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Kaon Condensation in "Nuclear Star" Matter

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ABSTRACT

The critical density for kaon condensation in "nuclear star" matter is computed up to two-loop order *in medium* (corresponding to next-to-next-to-leading order in chiral perturbation theory in free space) with a heavy-baryon effective chiral Lagrangian whose parameters are determined from KN scattering and kaonic atom data. To the order considered, the kaon self-energy has highly non-linear density dependence in dense matter. We find that the four-Fermi interaction terms in the chiral Lagrangian play an important role in triggering condensation, predicting for "natural" values of the four-Fermi interactions a rather low critical density, $\rho_c < 4\rho_0$.

In a recent paper, Brown and Bethe [1] suggested that if kaon condensates develop at relatively low matter density in the collapse of large stars, then low-mass black holes are more likely to form than neutron stars of the mass greater than 1.5 times the solar mass $M_\odot$. Ever since the seminal paper of Kaplan and Nelson[2], there have been numerous investigations on kaon condensation in dense neutron star matter as well as in nuclear matter based both on effective chiral Lagrangians[3, 4, 5, 6, 7] and on phenomenological off-shell meson-nucleon interactions[8, 9]. The results have been quite confusing: While the chiral Lagrangian calculations generally predict a relatively low critical density, $\rho_c \sim (2-4)\rho_0$, the phenomenological approaches have indicated that a kaon condensation at such a low density may be incompatible with kaon-nucleon data and in some versions seem to exclude any condensation at al. It is now understood [10, 11] that the main difference in the two approaches lies in terms higher than linear in density in the energy density of the matter.

In this paper, we report the first higher-order chiral perturbation calculation of the critical density with a chiral Lagrangian that when calculated to one loop order (that is to $\mathcal{O}(Q^2)$ relative to the leading order), correctly describes s-wave kaon-nucleon amplitude near threshold *and* that includes four-Fermi interactions constrained by kaonic atom data. Our prediction for critical density is $\rho_c \approx (3-4)\rho_0$.

To implement spontaneously broken chiral symmetry in the computation, we take the Jenkins-Manohar heavy-baryon chiral Lagrangian [12] as extended in [13] to $\mathcal{O}(Q^3)$ to describe s-wave kaon nucleon scattering to one loop order in chiral perturbation theory (ChPT). In addition to the usual octet and decuplet baryons and the octet pseudo-Goldstone fields, the $\Lambda(1405)$ was found to figure importantly in the kaon-nucleon process. This field which provides repulsion at threshold in K^-p scattering was introduced in [13] as an elementary field. By fitting the coefficients of the resulting chiral Lagrangian by empirical kaon nucleon s-wave scattering data at low energy, it was shown there that higher order chiral corrections can systematically be calculated while preserving the "naturalness" condition for on-shell scattering amplitudes. By a straightforward off-shell extension, we have predicted an off-shell kaon-nucleon amplitude that could be applied to kaonic atom [14] as well as kaon condensation phenomena. The predicted off-shell amplitude was found to be in fair agreement with the phenomenological fit obtained by Steiner [15]. A simple way of understanding the result so obtained is to use the chiral counting appropriate for the meson-baryon system. In heavy-baryon formalism (HBF), we can order the relevant observables as a power series in Q, say,

Q^ν where Q is the characteristic energy or momentum scale we are interested in and ν an integer. Thus to leading order, the kaon-nucleon amplitude T^{KN} goes as $\mathcal{O}(Q^1)$, to next order but involving only tree graphs as $\mathcal{O}(Q^2)$ and to next-to-next order (or N^{2}LO) at which one-loop graphs enter as $\mathcal{O}(Q^3)$. The off-shell amplitude calculated in [13] contains therefore all terms up to $\mathcal{O}(Q^3)$. This amplitude could be used in impulse approximation for in-medium processes. The corresponding contribution to the kaon self-energy given by Fig.1a is

$$\Pi_K^{imp}(\omega) \;\; = \;\; -\Big(\rho_p \mathcal{T}_{free}^{K^- p}(\omega) + \rho_n \mathcal{T}_{free}^{K^- n}(\omega)\Big) \tag{1}$$

where $\mathcal{T}^{KN}$ is the off-shell s-wave KN transition matrix [#1]. This can provide an optical potential for kaonic atom and the linear density approximation for kaon condensation. We will shortly discuss what critical density is obtained in this approximation.

To go beyond the linear density approximation as required for a more reliable treatment of both kaon condensation and kaonic atom, we need to compute the *effective action* (or *effective potential* for uniform matter). For this the first obvious correction to the self-energy (1) comes from the influence of the medium on the amplitude $\mathcal{T}^{\mathcal{KN}}$ which is readily taken into account by replacing the heavy nucleon propagator in Figs.1b-1f by the in-medium one

$$G^0(k) \simeq \frac{i}{v\cdot k + i\epsilon} - 2\pi\delta(k_0)\theta(k_F - |\vec{k}|) \tag{2}$$

where k_F is the nucleon Fermi momentum related to density ρ_N by the usual relation $\rho_N = \frac{\gamma}{6\pi^2} k_{F_N}^3$ with the degeneracy factor $\gamma = 2$ for neutron and proton in nuclear matter. We shall call this class of corrections $\delta\mathcal{T}_{\rho N}^{K^- N}$. It is clear that it will give rise to non-linear density dependence. Furthermore we expect it to be repulsive as it corresponds to the Pauli exclusion effect.

The second correction – which is a lot more important – comes from "particle-hole" excitations that do not figure in KN scattering but can contribute importantly in medium. These are depicted in Figs.2. Since we are dealing with s-wave kaon interaction, the most important configuration that K^- can couple to is the $\Lambda(1405)$ particle-nucleon hole (denoted as ΛN^{-1} with N either a proton (p) or neutron (n)). Thus Fig. 2a involves the ΛN^{-1}-ΛN^{-1} interaction whereas Figs. 2b can involve both the ΛN^{-1}-ΛN^{-1} and NN^{-1}-NN^{-1} interactions. In what follows we will not specifically consider the latter which involves no strangeness flavor: We will assume it to be given

[#1] The amplitude $\mathcal{T}^{KN}$ taken on-shell, *i.e.*, $\omega = M_K$, and the scattering length a^{KN} are related by $a^{KN} = \frac{1}{4\pi(1+M_K/m_B)} T^{KN}$.

by what is determined in the non-strange (nuclear) sector, *i.e.*, symmetry energy etc[#2]. Now for the s-wave in-medium kaon self-energy, the relevant four-Fermi interactions that involve a $\Lambda(1405)$ can be reduced to a simple form involving two unknown constants

$$\mathcal{L}_{4-fermion} \quad = \quad C_\Lambda^S \bar{\Lambda}\Lambda \mathrm{Tr}\bar{B}B + C_\Lambda^T \bar{\Lambda}\sigma^k \Lambda \mathrm{Tr}\bar{B}\sigma^k B \tag{3}$$

where B is the baryon (here nucleon) field, $C_\Lambda^{S,T}$ are the dimension-2 (M^{-2}) parameters to be fixed empirically and σ^k acts on baryon spinor.

Additional (in-medium) two-loop graphs that involve ΛN^{-1} excitations are given in Fig. 2c. They do not however involve contact four-Fermi interactions, so are calculable unambiguously.

We shall denote the sum of these contributions from Figs. 2 to the self-energy by Π_Λ [#3].

If one is only interested in critical density for kaon condensation and in properties of kaonic atom, it suffices to compute the kaon self-energy. For the equation of state to which we will return in a separate publication, nonlinearities in the condensate fields will of course have to be taken into account. This can be done in a straightforward way as described in [6]. Putting all graphs up to two loops in medium, we have the complete in-medium two-loop kaon self-energy

$$\Pi_K(\omega) \quad = \quad -\Big(\rho_p \mathcal{T}_{free}^{K^- p}(\omega) + \rho_n \mathcal{T}_{free}^{K^- n}(\omega)\Big) - \Big(\rho_p \delta\mathcal{T}_{\rho_N}^{K^- p}(\omega) + \rho_n \delta\mathcal{T}_{\rho_N}^{K^- n}(\omega)\Big) + \Pi_\Lambda(\omega) \tag{4}$$

where $\mathcal{T}_{free}^{K^- N}$ is the scattering amplitude obtained in LJMR [13], ρ_N ($N = p, n$) is the nucleon

[#2] In the sense of Fermi liquid, this part of interactions should be given in terms of the standard Landau-Migdal interactions which we assume, as in condensed matter physics [16], to be a fixed point theory. As such, one can take this part of interactions to be accurately given by nuclear matter properties.

[#3] There is one class of potentially important in-medium two-loop graphs that we have not taken into account in this paper and that is the set of graphs which could screen the leading $\mathcal{O}(Q)$ term associated with the vector-meson (ω and ρ) exchange [13]. To be specific, one can visualize it as Fig. 1a with the box replaced by an ω-exchange vertex and with the nucleon loop attached to another nucleon loop through a four-Fermi interaction. In the ω-exchange channel, the screening will go like $\sim \frac{1}{1+F_0}$ where the F_0 is the Landau-Migdal parameter [17]. In dense matter, one expects $F_0 > 0$. Hence one might fear that there could be substantial loss of attraction. We believe this will not be serious here for two reasons. Firstly the gauge coupling of the vector meson is predicted to scale *down* as density increases as in the hidden-gauge symmetry Lagrangian theory [18] so that F_0 will increase less rapidly than in the absence of the scaling. Secondly the vector-exchange attraction is scaled *up* by the BR scaling [19] by the factor $(m_\rho/m_\rho^\star)^2 > 1$ where m_ρ is the vector-meson mass with the star indicating the in-medium quantity. These two opposing effects are expected to more or less cancel out. We leave both effects out in this paper. They will be treated in detail in a longer paper in preparation.

density and $\delta\mathcal{T}_{\rho_N}^{K^-N}$ are the medium modifications, by Figs.1b-1f, to $\mathcal{T}_{free}^{K^-N}$ #4 and

$$\begin{aligned}\Pi_\Lambda(\omega) &= -\frac{\bar{g}_\Lambda^2}{f^2}\bar{g}_\Lambda^2\left(\frac{\omega}{\omega+m_B-m_\Lambda}\right)^2\left\{C_\Lambda^S\rho_p\left(\rho_n+\frac{1}{2}\rho_p\right)-\frac{3}{2}C_\Lambda^T\rho_p^2\right\}\\ &\quad -\frac{\bar{g}_\Lambda^4}{f^4}\rho_p\left(\frac{\omega}{\omega+m_B-m_\Lambda}\right)^2\omega^2\left(\Sigma_K^p+\Sigma_K^n\right)\end{aligned} \tag{5}$$

where $\bar{g}_\Lambda$ is the renormalized $KN\Lambda(1405)$ coupling constant determined in [13] and Σ_K^N is given by,

$$\Sigma_i^N(\omega) = \frac{1}{2\pi^2}\int_0^{k_{F_N}} d|\vec{k}|\frac{|\vec{k}|^2}{\omega^2-m_i^2-|\vec{k}|^2}. \tag{6}$$

In eq.(5), the first term comes from the diagrams of Figs. 2a and 2b and the second term from the diagrams of Figs. 2c. While the second term gives repulsion corresponding to a Pauli quenching, the first term can give either attraction or repulsion depending on the sign of $(C_\Lambda^S[\rho_n+\frac{1}{2}\rho_p]-\frac{3}{2}C_\Lambda^T\rho_p)$ with the constants $C_\Lambda^{S,T}$ being the only parameters that are not determined by on-shell scattering data. We can fix them from kaonic atom data [14] which require that there be an *effective attraction*. In condensed matter physics[16], such an attractive four-Fermi interaction in a particular kinematic situation turns by renormalization into a "marginally relevant" interaction that causes instability of the system, *e.g.*, pair condensation in superconductivity. It is tempting to conjecture that something similar happens here, leading to a strangeness condensation. However what appears to be different here from a generic case in condensed matter physics is that *the four-Fermi interaction contribution (5) has a quadratic ω dependence which makes it increasingly less effective toward the regime of small ω where the condensation takes place.*

We have now all the ingredients needed to calculate the critical density. For this, we will follow the procedure given in [6]. As argued in [3], we need not consider pions when electrons with high chemical potential can trigger condensation through the process $e^- \to K^-\nu_e$. Thus we can focus on the spatially uniform condensate

$$\langle K^-\rangle = v_K e^{-i\mu t} \tag{7}$$

where μ is the chemical potential which is equal, by Baym's theorem [20], to the electron chemical potential. We shall parametrize the proton and neutron densities by the proton fraction x and the nucleon density $u=\rho/\rho_0$ as

$$\rho_p = x\rho\ ,\ \ \rho_n=(1-x)\rho\ ,\ \ \rho = u\rho_0. \tag{8}$$

[#4]The explicit formulae will be given in a longer paper in preparation.

Then the energy density $\tilde{\epsilon}$ – which is related to the effective potential in the standard way – is given by,

$$\begin{aligned}\tilde{\epsilon}(u,x,\mu,v_K) &= \frac{3}{5}E_F^{(0)}u^{\frac{5}{3}}\rho_0 + V(u) + u\rho_0(1-2x)^2S(u) \\ &\quad -[\mu^2 - M_K^2 - \Pi_K(\mu,u,x)]v_K^2 + \mathcal{O}(v_K^3) \\ &\quad +\mu u\rho_0 x + \tilde{\epsilon}_e + \theta(|\mu| - m_\mu)\tilde{\epsilon}_\mu \end{aligned} \tag{9}$$

where $E_F^{(0)} = \left(p_F^{(0)}\right)^2/2m_N$ and $p_F^{(0)} = (3\pi^2\rho_0/2)^{\frac{1}{3}}$ are, respectively, Fermi energy and momentum at nuclear density. The $V(u)$ is a potential for symmetric nuclear matter as described in [21] which we believe is subsumed in contact four-Fermi interactions (and one-pion-exchange – nonlocal – interaction) in the non-strange sector as mentioned above. It will affect the equation of state in the condensed phase but not the critical density, so we will drop it from now on. The nuclear symmetry energy $S(u)$ – also subsumed in four-Fermi interactions in the non-strange sector – does play a role as we know from ref.[21]: Protons enter to neutralize the charge of condensing K^-'s making the resulting compact star "nuclear" rather than neutron star as one learns in standard astrophysics textbooks. We take the form advocated in [21]

$$S(u) = \left(2^{\frac{2}{3}} - 1\right)\frac{3}{5}E_F^{(0)}\left(u^{\frac{2}{3}} - F(u)\right) + S_oF(u) \tag{10}$$

where $F(u)$ is the potential contributions to the symmetry energy. Three different forms of $F(u)$ were used in [21]:

$$F(u) = u\ ,\ \ F(u) = \frac{2u^2}{1+u}\ ,\ \ F(u) = \sqrt{u}. \tag{11}$$

It will turn out that the choice of $F(u)$ does not significantly affect the critical density.

The contributions of the filled Fermi seas of electrons and muons are [6]

$$\begin{aligned}\tilde{\epsilon}_e &= -\frac{\mu^4}{12\pi^2} \\ \tilde{\epsilon}_\mu &= \epsilon_\mu - \mu\rho_\mu \\ &= \frac{m_\mu^4}{8\pi^2}\left((2t^2+1)t\sqrt{t^2+1} - \ln(t^2+\sqrt{t^2+1})\right) - \mu\frac{p_{F_\mu}^3}{3\pi^2}\end{aligned} \tag{12}$$

where $p_{F_\mu} = \sqrt{\mu^2 - m_\mu^2}$ is the Fermi momentum and $t = p_{F_\mu}/m_\mu$. The ground-state energy prior to kaon condensation is obtained by extremizing $\tilde{\epsilon}$ with respect to x, μ and v_K:

$$\left.\frac{\partial\tilde{\epsilon}}{\partial x}\right|_{v_K=0} = 0\ ,\ \ \left.\frac{\partial\tilde{\epsilon}}{\partial\mu}\right|_{v_K=0} = 0\ ,\ \ \left.\frac{\partial\tilde{\epsilon}}{\partial v_K^2}\right|_{v_K=0} = 0 \tag{13}$$

$F(u)$	u	$\frac{2u^2}{1+u}$	$\sqrt{u}$
u_c	3.90	3.77	4.11

Table 1 : Critical density in linear density approximation corresponding to Fig. 1a

from which we obtain three equations corresponding, respectively, to beta equilibrium, charge neutrality and dispersion relation:

$$\begin{aligned} \mu &= 4(1-2x)S(u) \\ 0 &= -xu\rho_0 + \frac{\mu^3}{3\pi^2} + \theta(\mu - m_\mu)\frac{p_\mu^3}{3\pi^2} \\ 0 &= D^{-1}(\mu,u,x) = \mu^2 - M_K^2 - \Pi_K(\mu,u,x) \equiv \mu^2 - {M_K^\star}^2(\mu,u,x) \end{aligned} \tag{14}$$

where $p_\mu = \sqrt{\mu^2 - m_\mu^2}$. We have solved these equations using for the kaon self-energy (a) the linear density approximation, eq.(1) and (b) the full two-loop result, eq.(4). Table 1 shows the case (a) for different symmetry energies eq.(11). We see that the precise form of the symmetry energy does not matter quantitatively, so we will simply take $F(u) = u$ from now on. The corresponding "effective kaon mass" $M_K^\star$ is plotted vs. u in Fig. 3 in solid line. Note that even in this linear density approximation kaon condensation *does* take place, *albeit* at a bit higher density than obtained before.

We now turn to the (in-medium) two-loop calculation. For this we need to fix the parameters $C_\Lambda^{S,T}$. This could be done with kaonic atom data. The presently available data [14] imply that the optical potential for the K^- in medium has an attraction of the order of

$$\Delta V \approx -(180 \pm 20) \ \ \text{MeV} \ \ at \ \ u = 0.97 \tag{15}$$

which implies approximately for $x = 1/2$

$$(C_\Lambda^S - C_\Lambda^T) f_\pi^2 \approx 10. \tag{16}$$

This leaves one parameter free for $x \neq 1/2$. It will, however, turn out that this freedom does not diminish significantly the predictiveness of the theory. We shall not attempt to fine-tune these constants in this work but take (16) to be some mean value in what follows. The result turns out be pretty much insensitive to the precise values of the constants. When one can pin them down

u	M_K^*	ΔV	x	$-\rho\mathcal{T}^{free}$	$-\rho\delta\mathcal{T}^{free}$	Π_Λ^1	Π_Λ^2
0.2	424.6	−70.37	0.5	−0.0673	0.0034	−0.1998	0.007607
0.4	390.0	−105.0	0.5	−0.0920	0.0084	−0.3024	0.006641
0.6	364.3	−130.7	0.5	−0.1173	0.0143	−0.3610	0.005635
0.8	342.6	−152.4	0.5	−0.1462	0.0207	−0.3996	0.004794
1.0	323.5	−171.5	0.5	−0.1789	0.0274	−0.4250	0.004088
1.2	306.2	−188.8	0.5	−0.2150	0.0343	−0.4404	0.003483
1.4	289.9	−205.1	0.5	−0.2540	0.0413	−0.4460	0.002945

Table 2 : Self-energies for kaonic atom in nuclear matter ($x = 0.5$) in unit of M_K^2 for $(C_\Lambda^S - C_\Lambda^T)f_\pi^2 = 10$ and $F(u) = u$. $\Delta V \equiv M_K^\star - M_K$ is the attraction (in unit of MeV) at given density.

(through, *e.g.*, isotope effects) from kaonic atom data, our prediction could be made considerably more precise.

In Table 2, we list the predicted density dependence of the real part of the kaonic atom potential for $x = 0.5$ obtained for $(C_\Lambda^S - C_\Lambda^T)f_\pi^2 \approx 10$. To exhibit the role of $\Lambda(1405)$ in the kaon self-energy, we list each contribution of Π. Here $\Pi_{free} = -\rho_N \mathcal{T}_{free}^{K^-N}$, $\delta\Pi = -\rho_N \delta\mathcal{T}^{K^-N}$, Π_Λ^1 corresponds to the first term of eq.(5) which depends on $C_\Lambda^{S,T}$ and Π_Λ^2 to the second term independent of $C_\Lambda^{S,T}$. We observe that the $C_\Lambda^{S,T}$-dependent term plays a crucial role for attraction in kaonic atom. For the value that seems to be required by the kaonic atom data, (16), the critical density comes out

$C_\Lambda^S f_\pi^2$	10	5	0
u_c	3.13	3.33	3.69

Table 3 : Critical density u_c in in-medium two-loop chiral perturbation theory for $(C_\Lambda^S - C_\Lambda^T)f_\pi^2 = 10$ and $F(u) = u$.

to be about $u_c \approx 3$, rather close to the original Kaplan-Nelson value.

In Table 3 and Fig. 3 are given the predictions for a wide range of values for $C_\Lambda^S f_\pi^2$. What is remarkable here is that while the $C_\Lambda^{S,T}$-dependent four-Fermi interactions are *essential* for triggering kaon condensation, the critical density is quite insensitive to their strengths. In fact, reducing the constant $(C_\Lambda^S - C_\Lambda^T)f_\pi^2$ that represents the kaonic atom attraction by an order of magnitude to 1 with $C_\Lambda^S f_\pi^2 = 10,\ 0$ modifies the critical density only to $u_c \approx 3.3,\ 4.5$, respectively.

In conclusion, we have shown that chiral perturbation theory at order N^{2}LO predicts kaon condensation in "nuclear star" matter at a density $\rho_c < 4\rho_0$ with a large fraction of protons – $x = 0.1 \sim 0.2$ at the critical point and rapidly increasing afterwards – neutralizing the negative charge of the condensed kaons. For this to occur, four-Fermi interactions involving $\Lambda(1405)$ are found to play an important role in triggering the condensation but the critical density is surprisingly insensitive to the strength of the four-Fermi interaction. This makes the condensation phenomenon even more robust than thought before. We have not taken into account the BR scaling [19] – which we should, to be fully consistent. An approximate account of the BR scaling is found to lower the critical density below $3\rho_0$, practically independently of the magnitude of $C_\Lambda^{S,T}$. As remarked above, there are compensating effects such as the screening of the vector channel which has to be considered on the same footing, so a fully consistent treatment would require more work. What is fairly certain is that although the detailed mechanism appears quite different here from that in the low-order treatments, once the condensation sets in, the rest of the star properties are expected to resemble closely the structure obtained in [6, 7]. A detailed analysis of the "nuclear star" that we obtained in this paper will be made elsewhere.

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Figure Captions

- **Figure 1:** (a): The linear density approximation to the kaon self-energy in medium, Π_K. The square blob represents the off-shell K^-N amplitude calculated to $\mathcal{O}(Q^3)$; (b)-(f): medium corrections to $\mathcal{T}^{KN}$ of fig.(a) with the free nucleon propagator indicated by a double slash replaced by an in-medium one, eq.(2). The loop labeled ρ_N represents the in-medium nucleon loop proportional to density, N^{-1} the nucleon hole (n^{-1} and/or p^{-1}), the external dotted line stands for the K^- and the internal dotted line for the pseudoscalar octets π, η, K.

- **Figure 2:** Two-loop diagrams involving $\Lambda(1405)$ contributing to the kaon self-energy. Diagrams (a) and (b) involve four-Fermi interactions describing the ΛN^{-1}-ΛN^{-1} vertex. Diagram (c) does not involve four-Fermi interactions and hence is unambiguously determined by on-shell parameters. Here the internal dotted line represents the kaon.

- **Figure 3:** Plot of the effective kaon mass $M_K^\star$ obtained from the dispersion formula $D^{-1}(\mu, u) = 0$ vs. the chemical potential μ prior to kaon condensation, with $F(u) = u$. The solid line corresponds to the linear density approximation and the dashed lines to the in-medium two-loop results for $(C_\Lambda^S - C_\Lambda^T) f_\pi^2 = 10$ and $C_\Lambda^S f_\pi^2 = 10, 5, 0$ respectively from the left. The point at which the chemical potential μ (denoted μ_{cp}) intersects $M_K^\star$ corresponds to the critical point.

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